THERMOCONVECTIVE MOTION IN A VOLUME CONTAINING TWO LAYERS OF A VISCOUS LIQUID

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Convective motion is considered in a system of two liquids filling a rectangular cavity with a free surface.

A knowledge of mechanisms of convective heat and mass transfer in systems comprising several immiscible liquids is of interest for many branches of science and technology. Investigation of convection in multilayer systems is encouraged, on the one hand, by developing intensification methods for convective agitation in a liquid (chemical technology), and on the other, by searching suppression methods for convective agitation (space study of materials) by the corresponding selection of configuration and parameters for liquid layers.

The presence of temperature gradients in a layered liquid found in a gravity field may cause a complicated convective motion due to the action of volume thermogravitational and surface thermocapillary forces. The existence of tangential surface tension forces at the interfaces (liquid–liquid, liquid–gas) can have a substantial effect on the heat and mass transfer in the liquid. One of the problems arising in the analysis of such a motion is a comparative estimate for the role of thermocapillary and thermogravitational convection [1-3], and capillary forces at the interface and on a free surface.

1. Statement of the problem. We will consider convective motion in a system of two liquids filling a rectangular cavity with a free upper boundary. The lateral walls are maintained at different temperatures T_1 and T_2, the bottom and the free surface are thermally insulated. It is assumed that the liquids do not mix, and the liquid density in the upper layer is smaller than in the lower one ($\rho_1 > \rho_2$) (Fig. 1). Deformation of both the free surface and the liquid–liquid interface are considered to be negligible.

A mathematical formulation of the problem includes the Navier–Stokes continuity and heat conduction equation written for each layer of the liquid. With the vortex ω, the stream function ψ, and the temperature T chosen as the unknown functions and going to dimensionless variables, we will write the assumed equations:

in the lower layer:

$$\frac{\partial \omega_1}{\partial t} + u_1 \frac{\partial \omega_1}{\partial x} + v_1 \frac{\partial \omega_1}{\partial y} = \mathrm{Pr}_1 \nabla^2 \omega_1 + \mathrm{Gr}\,(\mathrm{Pr}_1)^2 \frac{\partial T_1}{\partial x},$$
$$\frac{\partial T_1}{\partial x} + u_1 \frac{\partial T_1}{\partial x} + v_1 \frac{\partial T_1}{\partial y} = \nabla^2 T_1, \qquad (1)$$
$$\nabla^2 \psi_1 = \omega_1;$$

in the upper layer:

$$\frac{\partial \omega_2}{\partial t} + u_2 \frac{\partial \omega_2}{\partial x} + v_1 \frac{\partial \omega_2}{\partial y} = \mathrm{Pr}_2 \nabla^2 \omega_2 + \mathrm{Gr}\,\mathrm{Pr}_1^2 \beta \frac{\partial T_2}{\partial x},$$
$$\frac{\partial T_2}{\partial t} + u_2 \frac{\partial T_2}{\partial x} + v_2 \frac{\partial T_2}{\partial y} = \nabla^2 T_2, \qquad (2)$$
$$\nabla^2 \psi_2 = \omega_2.$$

Here

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$$\omega=\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}, \quad u=\frac{\partial \psi}{\partial y}, \quad v=-\frac{\partial \psi}{\partial x},$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2}.$$

The boundary conditions on the lateral walls (x = 0, x = 1) and at the bottom of the dish (y = 0) are the liquid adhesion conditions

$$\psi_i=0, \ \partial\psi_i/\partial n=0. \tag{3}$$

On the free surface (y = 1), beside the no-flow condition, the balance condition for viscous and thermocapillary forces (the Marangoni effect) is specified

$$\psi=0, \ \frac{\partial u_2}{\partial y}=\mathrm{Ma}_2\frac{\eta_1}{\eta_2}\frac{\partial T_2}{\partial x}, \quad \mathrm{Ma}_2=-\frac{d\sigma_2}{dT}\frac{H(T_1-T_0)}{\varkappa_1\rho_1\nu_1}. \tag{4}$$

The bottom and the free surface are considered thermally insulated

$$\partial T_i/\partial n=0, \ y=0, \ y=1. \tag{5}$$

At the liquid–liquid interface, y = 1/2, we have specified:
the continuity of the temperature and the horizontal velocity component

$$T_1=T_2, \ \left(\frac{\partial\psi}{\partial y}\right)_1=\left(\frac{\partial\psi}{\partial y}\right)_2; \tag{6}$$

the impermeability conditions, the interface is undeformable:

$$\psi_1=\psi_2=0; \tag{7}$$

the continuity of the heat flux

$$k_1\left(\frac{\partial T}{\partial y}\right)_1=k_2\left(\frac{\partial T}{\partial y}\right)_2; \tag{8}$$

the balance of viscous forces

$$\frac{\partial u_1}{\partial y}=\eta\frac{\partial u_2}{\partial y}+\mathrm{Ma}_1\frac{\partial T_1}{\partial x},$$

or in other variables

$$\omega_1=\eta\omega_2+\mathrm{Ma}_1\frac{\partial T_1}{\partial x}, \quad \mathrm{Ma}_1=-\frac{d\sigma_1}{dT}\frac{H(T_1-T_0)}{\varkappa_1\rho_1\nu_1}. \tag{9}$$

From the conditions at the interface (6)-(9) it can be seen that the vortex is lacking yet another condition.

In this case we will take advantage of the analogy with boundary conditions near a solid surface. We will expand the stream function at both sides from the liquid–liquid interface into Taylor's series

$$\psi_i=\psi_{i,0}+\frac{\partial\psi_i}{\partial y}h_i+\frac{\partial^2\psi_i}{\partial y^2}h_i^2+O(h^3),$$

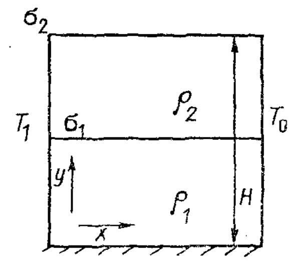

Fig. 1. Geometry of the problem

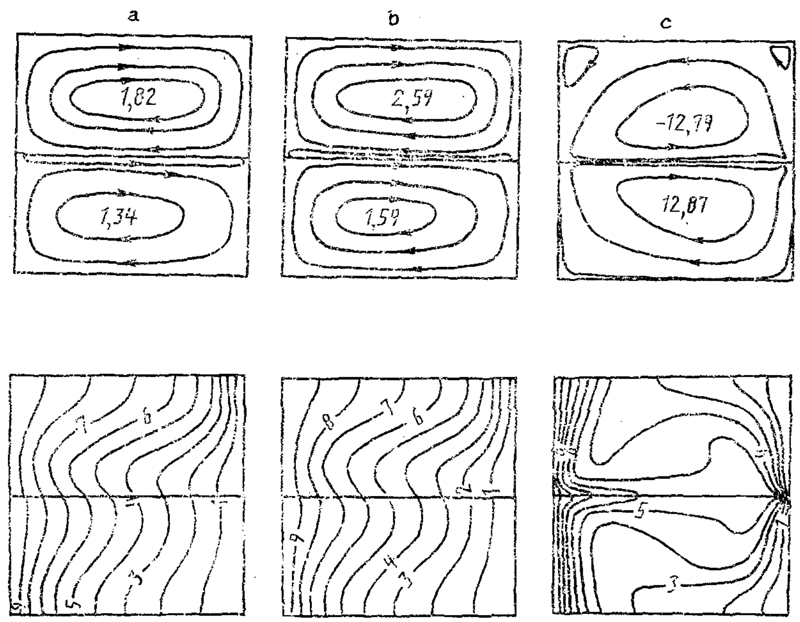

Fig. 2. Streamlines and isotherms for a closed system $Ma_2 = 0$, $Gr = 10^4$: a) natural convection, $Ma_1 = 0$; b) $Ma_1 = 10^2$; c) $Ma_1 = 10^3$.

we will substitute the values of $(\partial\psi_i/\partial y)$ found into Eq. (6), taking into account that in the case of an undeformable interface $\omega_i = \partial^2\psi_i/\partial y^2$ and $\psi_i = 0$ for $y = 1/2$, we obtain a missing equation for the vortex

$$(\omega_1 + \omega_2) = \frac{2}{h^2}(\psi_1 + \psi_2). \qquad (10)$$

Hereinafter the index $i = 1$ refers to the lower layer, the index $i = 2$ to the upper one.

In formulas (1)-(10) use is made of the dimensionless variables based on the parameters and physical properties of the lower liquid: $x = x'/H$, $y = y'/H$ are the coordinates, $u = u'H/\kappa_1$ is the horizontal velocity component, $v = v'H/\kappa_1$ is the vertical velocity component, $\omega = \omega'H^2/\kappa_1$ is the vortex, $\psi = \psi'\kappa_1 H$ is the stream function, κ_1 is the thermal diffusivity, $Pr_1 = \nu_1/\kappa_1$ and $Pr_2 = \nu_2/\kappa_2$ are the Prandtl numbers, $k = k_2/k_1$, $\kappa = \kappa_2/\kappa_1$, $\beta = \beta_2/\beta_1$, and $\eta = \eta_2/\eta_1$ are the ratios of the thermal conductivities, thermal diffusivities, viscosity factors, and coefficients of thermal expansion, respectively, $Gr = \beta_1 g\Delta TH^3/\nu_1^2$ is the Grashof number.

For the numerical solution of the problem use was made of the implicit difference circuit on a nonuniform grid with a monotone approximation of convective terms [4]. The major results were obtained on the grid with the number of nodes 51 × 51. The finest spatial pitches were employed near the interface and close to the free surface. In numerical simulation the development of the temperature and vortex fields from the quiescent state to the steady one is traced and a large body of information is obtained on liquid motion in a two-layer system. This work presents the results solely for the steady-state regime.

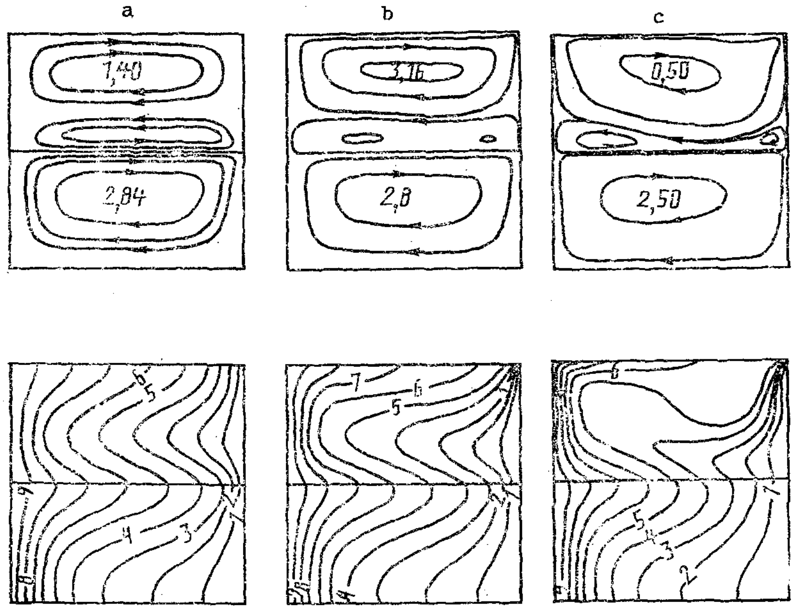

Fig. 3. Current lines and isotherms in an open system: a) absence of the Marangoni force at the upper boundary, $Ma_2 = 0$, $u_x = 0$; b) $Ma_2 = 10^3$; c) $Ma_2 = 5 \cdot 10^3$.

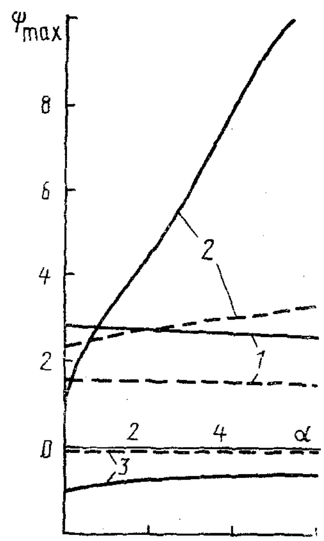

Fig. 4. Extreme values of the stream function in each vortex vs ratio of the Marangoni forces on the free surface and at the liquid–liquid interface $\alpha = Ma_2/Ma_1$.

In solving the problem (1)-(10) the dimensionless parameters are chosen so that the Prandtl number for the lighter liquid is twice as large as for the heavier one ($Pr_1 = 1$, $Pr_2 = 2$), and the dynamic viscosities are equal. The other parameters are chosen in the following manner: $k = 0.5$; $\rho = 0.8$; $\beta = 2$, $\kappa = 0.625$.

2. Results of the calculations. Figure 2 traces the development of convection in a closed volume with the upper solid boundary ($y = 1$, $u = v = 0$) as the interphase Marangoni number Ma_1 changes. The streamlines are presented in the upper portion of the figure (the maximal and minimal values for the stream function characterizing the intensity of motion are given), and the isotherms at the bottom.

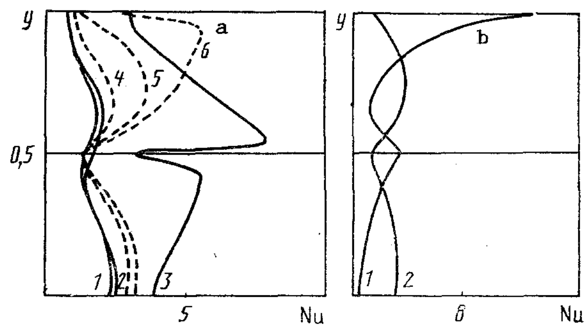

Fig. 5. Distribution of the local Nusselt number.

In the case of gravitational convection (Fig. 2a) ($Gr = 10^4$, $Ma_1 = Ma_2 = 0$), vortex motion with a positively directed circulation is formed in each layer. The motion intensity in the upper layer is somewhat larger than in the lower one. At the liquid–liquid interface a conflict situation arises and as a result of the interaction of two large convective cells in the lower layer a secondary flow of low intensity emerges with a negatively directed circulation $\psi_{min} = -0.03$.

Convective motion accelerates the transfer of heat from the heated wall, and the curve of the isotherms shows that the region in the vicinity of the upper boundary becomes the most heated.

With the emergence at the interface of even a small Marangoni force ($Ma_1 = 10^2$) a small vortex cell with a negative direction of circulation moves into the upper liquid. Figure 2b corresponds to $Gr = 10^4$, $Ma_1 = 10^4$, and $Ma_2 = 0$. The intensity of motion in both cases does not change considerably.

As the Marangoni force further increases at the liquid–liquid interface the cell with negative circulation grows, and the intensity of motion in it increases. A positively directed vortex in the upper layer contracts with decreasing intensity. The isotherm distribution pattern shows that with increasing Ma_1 the most intensive transfer of heat is observed in the vicinity of the interface; in the upper portion of the system the heat transfer decreases. For $Ma_1 = 10^4$, convection in the layered system is mainly determined by the Marangoni force, the role of natural convection being insignificant. Figure 2c corresponds to $Gr = 10^4$ and $Ma_2 = 0$. The vortex motion becomes almost symmetric about the interface.

Two large oppositely directed vortex cells are formed in the system. The intensity of motion in the upper and lower layers is approximately the same. The presence of gravitational force makes itself evident in the existence of two small positively directed cells of weak intensity in the upper corners of the system. The maximal rate of liquid motion along the interface is a factor of 10^2 higher than in the case of Fig. 2a.

The isotherm pattern confirms the predominant role of the Marangoni convection. The heat from a hot surface is carried away by a convective flow, and in the central region an isothermal zone forms. The region of the size $0.3 \le x \le 0.8$ and $0.25 \le y \le 0.75$ has a nearly uniform temperature $0.4 \le T \le 0.5$.

The next part of the work deals with revealing the action of Marangoni forces at the upper free boundary.

We will consider a variant when the upper boundary is open but it is not subject to the action of surface tension forces ($u_x = 0$, $y = 1$). Figure 3a corresponds to $Gr = 10^4$, $Ma_1 = 10^3$, and $Ma_2 = 0$. The vortex motion structure in this case is analogous to the case with the upper solid boundary for the given set of parameters, which has also been considered numerically but the isoline patterns are not presented here. The intensity of motion close to the upper boundary is nearly twice as large as for the free surface in comparison with the solid one. Conditions at the upper boundary have no substantial effect on the intensity of motion in the lower layer.

Under the action of the Marangoni force on the free surface (Fig. 3b, $Ma_1 = 10^3$, $Ma_2 = 10^3$, $Gr = 10^4$) the convective cell in the vicinity of the upper boundary slightly increases in size with considerably increasing intensity of motion in it. The presence of the Marangoni force at the liquid–liquid interface Ma_1 prevents the capillary force Ma_2 on the free surface from affecting the motion in the lower layer. As a result, the intensity of motion in the lower liquid practically does not change.

A variant when the Marangoni force at the upper boundary is larger than at the interface ($Ma_1 = 10^3$, $Ma_2 = 5 \cdot 10^3$, $Gr = 10^4$) is presented in Fig. 3c. As in the previous case, the rate of one-vortex motion in the lower layer and the heat transfer practically remain unchanged.

In the upper layer the positively directed cell near the surface increases in size, compressing the negatively directed one. The temperature in the center of the layer is almost uniform, with drastic differences being observed in the vicinity of the walls.

It should be noted that as the Marangoni force increases further on the free surface ($Ma_2 = 10^4$, $Ma_2/Ma_1 = 10$, $Gr = 10^4$) its effect is so dramatic that the action of the interface capillary force is suppressed and in the lower layer adjacent to the interface a small negatively directed cell emerges. In the process, as a series of calculations for $Ma_1 = 10^2$ has shown, with decreasing interphase force the emergence of the second vortex in the lower layer close to the boundary is observed when capillary forces differ less.

Figure 4 shows maximal and minimal values for the stream function in each vortex versus the ratio of capillary forces. Curves 2 and 3 correspond to the extreme values of ψ in the upper layer of each cell, curves 1 characterize one-vortex motion in the lower layer. Solid lines correspond to $Ma_1 = 10^3$, dashed ones to $Ma_1 = 10^2$.

As was previously mentioned, the mass transfer rate in the lower layer (curves 1) insignificantly decreases with varying α. The vortex motion force in the positively directed cell of the upper layer increases with increasing α (curves 2), the sharper the larger the Marangoni number is at the interphase.

In considering the convection problems, alongside the velocity field, a change in the Nusselt number depending on the parameters of the problem is of interest. In the given work we calculated local values of $Nu = (\partial T/\partial x)_{x=0}$ and an average value of $Nu = \int Nu\, dy$. In calculations for control over the attainment of a steady state use was made of the equality condition for the average Nusselt number value on the hot wall and on the cold one.

The distribution of the local Nusselt number with height on the hot wall is shown in Fig. 5a. Curves 1-6 correspond to the cases presented on the isoline patterns 2a, 2b, 2c; 3a, 3b, and 3c, respectively. To make it clearer, curves 4-6 are shown with dashed lines.

For natural convection (curve 1), the local maximum of the heat transfer rate is found close to the bottom of the system.

The minimum local Nusselt number forms at the liquid–liquid interface with the emergence of the interphase Marangoni force, the minimum becoming more pronounced as Ma_1 increases. It should be noted that the presence of the thermocapillary force Ma_1 affects the heat transfer both in the upper and lower liquids. As the comparison of curves 2 and 4 suggests the difference in the Nusselt number distribution in the open dish (Fig. 3a) and in the closed one (Fig. 2a) is insignificant.

The emergence of the Marangoni force at the upper boundary substantially increases the heat transfer in the upper layer (see Fig. 5a, curves 5 and 6). For large values of Ma_2 (variant 3c, curve 6 in Fig. 5a), an active transfer of heat is observed along the free surface. The structure of heat transfer in the lower liquid does not substantially change.

Figure 5b shows the Nusselt number distribution at the hot (curve 1) and cold (curve 2) boundaries for moderate Marangoni numbers (which corresponds to Fig. 3b).

As is evident from the figure, the rate of heat removal from the hot wall does not change abruptly with the dish height. Near the cold boundary the Nusselt number distribution has a more pronounced maximum. This maximum is attained near the free surface where, as follows from Fig. 3b, the isotherms "gather" together.

On the basis of the performed analysis of thermoconvective flows it may be inferred that for liquids, which are fairly close in physical properties, the motion in the lower layer is largely determined by the Marangoni force at the interface. The motion in the upper layer is considerably affected by both the interphase Marangoni force and capillary forces on the free surface.

NOTATION

x, y, coordinates; t, time; v, u, vertical and horizontal velocity components; ω, vortex; ψ, stream function; T, temperature; T_1 and T_0, temperatures of hot wall and cold one; L, length of dish; H, thickness of two layers; η_i, ν_i, k_i, κ_i, β_i, and ρ_i, dynamic and kinematic viscosity factors, thermal conductivity, thermal diffusivity, coefficients of thermal expansion and density, respectively; $\eta = \eta_2/\eta_1$, $\kappa = \kappa_2/\kappa_1$, $\beta = \beta_2/\beta_1$, and $k = k_2/k_1$, corresponding ratios of coefficients in two-layer system; $\sigma = \sigma(T) = \alpha T$, coefficient of surface tension; $Pr_i = \nu_i/\kappa_i$, Prandtl number; $Ma_i = d\delta_i(T_1 - T_0)/(dT\ \kappa_1\rho_1\nu_1)$, Marangoni number; $Gr = \beta_1 g(T_1 - T_0)H^3/\nu_1^2$, Grashof number; $a = H/L = 1$, layer length to layer thickness ratio. Indices: i=1 refers to lower liquid, i=2 to upper liquid,′ to corresponding dimensional variables.

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